

Magnetohydrodynamics (MHD) Forced Convective Flow and Heat Transfer Over a Porous Plate in a Darcy-Forchheimer Porous Medium in Presence of Radiation

A. K. Jhankal¹, R. N. Jat², Deepak Kumar³

¹Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun – 248 007, India; ^{2,3}Department of Mathematics, University of Rajasthan, Jaipur-302 004, India.

ABSTRACT

Aim and Objective: An analysis is made to study the problem of boundary layer forced convective flow and heat transfer of an incompressible fluid past a plate embedded in a Darcy-Forchheimer porous medium in presence of transverse magnetic field. Thermal radiation term is considered in the energy equation.

Methodology: The governing partial differential equations are transformed into self-similar ordinary differential equations using similarity transformations, which are then solved using Runge-Kutta fourth order method along with shooting technique.

Results: The numerical results are plotted in some figures and the variations in physical characteristics of the flow dynamics and heat transfer for several parameters involved in the equations are discussed. It is found that these parameters have significantly effects on the flow and heat transfer.

Conclusion: It is found that the influence of the physical parameters viz. Porous medium parameter, the Inertial parameter, the Magnetic parameter, the Prandtl number, and the Radiation parameter have significantly effects on the flow and heat transfer.

Key Words: MHD, Forced convective flow, Darcy-Forchheimer porous medium, Thermal radiation

NOMENCLATURE

B_0	Constant applied magnetic field	M	Magnetic parameter
C_p	Specific heat at constant pressure	N	Radiation parameter
f	Dimensionless stream function	Pr	Prandtl number
k	Darcy permeability of the porous medium	q_r	Radioactive heat flux
k'	Forchheimer resistance factor	S	Suction parameter.
k_1	Parameter of the porous medium	T	Temperature of the fluid
k_2	Inertial parameter	u, v	Velocity component of the fluid along the x and y directions, respectively
k^*	Absorption coefficient	x, y	Cartesian coordinates along the surface and normal to it, respectively

Corresponding Author:

A. K. Jhankal, Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun – 248 007, India;
 E-mail: anujjhankal@yahoo.com

ISSN: 2231-2196 (Print)

ISSN: 0975-5241 (Online)

DOI: <http://dx.doi.org/10.7324/IJCRR.2017.9114>

Received: 14.04.2017

Revised: 02.05.2017

Accepted: 17.05.2017

Greek symbols

η	Similarity variable
ρ	Density of the fluid
μ	Viscosity of the fluid
σ_e	Electrical conductivity
κ	Thermal conductivity
ν	Kinematic viscosity
θ	Dimensionless temperature
θ_r	Temperature ratio parameter

Superscript

'	Derivative with respect to η
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Subscripts

w	Properties at the plate
∞	Free stream condition

INTRODUCTION

The study of boundary layer flow behavior and heat transfer characteristics of a Newtonian fluid past a plate embedded in a fluid saturated porous medium is important because the analysis of such flows finds extensive applications in engineering processes, especially in the enhanced recovery of petroleum resources and packed bed reactors (Pal and Shrivakumara [1]). A better understanding of convection through porous medium can benefit several areas like geophysical flow problems, grain storage, catalytic reactor, insulation design, material handling conveyers, geothermal system, filtering devices etc. In recent years, MHD flow problem have become more important industrially. Indeed, MHD boundary layer flow behavior and heat transfer characteristics of a Newtonian fluid past a vertical plate embedded in a fluid saturated porous medium is significant type of flow having considerable practical application in engineering processes. The MHD boundary layer flow of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. Hydromagnetic free convection flows have great significance for the applications in the fields of stellar and planetary magnetospheres, aeronautics. Engineers apply MHD principle in the design of heat exchangers, pumps, in space vehicle propulsion, thermal protection, control and re-entry and in creating novel power generating systems. However, hydromagnetic flow and heat transfer problems have become more important industrially. In many metallurgical processes involve the cooling of many continuous strips or filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled.

Convective heat transfers with thermal radiation are very important in the process involving high temperature such as gas turbines, nuclear power plant and thermal energy storage etc. in light of these various applications, Hossain and Takhar [2] studied the effect of thermal radiation using Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Furthermore Hossain et al. [3-4] studied the thermal radiation of a grey fluid which is emitting and absorbing radiation in a non-scattering medium. Free convective flows in a saturated porous medium were made by Cheng and Minkowycz [5] and Cheng [6]. Wilks [7] discussed the combined forced and free convection flow along a semi-infinite plate extending vertically upwards. Lai and Kulacki [8] investigated the coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium. Furthermore, some important contribution in Darcian and non-Darcian mixed convection about a vertical plate were made by Hsu and Cheng [9], Vafai and Tien [10]. Soundalgekaret et al. [11] discussed the combined free and forced convection flow past a semi-infinite plate with variable surface temperature. The problem of Darcy-Forchheimer mixed convection heat and mass transfer in fluid-saturated porous media was studied by Rami et al. [12]. Goren [13] was one of the first to study the role of thermophoresis in the laminar flow of a viscous and incompressible fluid. Most previous studies of the same problem neglected viscous dissipation and thermophoresis. But Gebhart [14] has shown that the viscous dissipation effect plays an important role in natural convection in various devices that are subjected to large variations of gravitational force or that operate at high rotational speeds. Analytically study of the non-Darcian effects on a vertical plate natural convection in porous media was made by Hong et al. [15]. Kaviany [16] studied the Darcy-Brinkman model to study the effects of boundary and inertia forces on forced convection over a fixed impermeable heated plate embedded in a porous medium. Chen and Ho [17] discussed the effects of flow inertia on vertical, natural convection in saturated porous media. Kumari et al. [18] investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium. Anghel et al. [19] studied the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. The effects of magnetic field and thermal radiation on forced convection flow were made by Damesh et al. [20]. Samad and Rahman [21] investigated the thermal radiation interaction with unsteady MHD boundary layer flow past a continuous moving vertical porous plate immerse in a porous medium with time dependent suction and temperature, in presence of magnetic field with radiation. Postelnicu [22] analyzed the influence of magnetic field on heat and mass transfer from vertical surfaces in porous

media considering Soret and Dufour effects. Srinivasacharya and Upendar [23] examined the free convection in MHD micropolar fluid under the influence of Dufour and Soret effects. Duwairi and Damesh [24] studied the natural convection heat and mass transfer by steady laminar boundary layer flow over an isothermal vertical flat plate embedded in a porous medium. Mukhopadhyay et al. [25] investigated the forced convection flow and heat transfer in a porous medium using the Darcy-Forchheimer model.

A new dimension is added to the abovementioned study by considering the effects of Darcy-Forchheimer porous media. It is well known that Darcy's law is an empirical formula relating the pressure gradient, the bulk viscous fluid resistance and the gravitational force for a forced convective flow in a porous medium. Deviations from Darcy's law occur when the Reynolds number based on the pore diameter is within the range of 1 to 10 (Ishak et al. [26]). For flow through porous medium with high permeability, Brinkman [27] as well as Chen et al. [28] argue that the momentum equation must reduce to the viscous flow limit and advocate that classical frictional terms be added in Darcy's law. Mishra and Jena [29] find numerical Solution of MHD flow with viscous dissipation. Vidyasagar et al. [30] discussed heat and mass Transfer effects on MHD boundary layer flow over a moving vertical porous plate. Mukhopadhyay et al. [31] investigated forced convective flow and heat transfer over a porous plate in a Darcy-Forchheimer porous medium in presence of radiation. Kankanala and Bandari [32] studied mixed convection flow of a casson fluid over an exponentially stretching surface with the effects of soret, dufour, thermal radiation and chemical reaction in presence of magnetic field. Jena [33] discussed numerical solution of boundary layer flow with viscous dissipation in presence of magnetic field. Venkateswarlu et al [34] studied effects of chemical reaction and heat Generation on boundary layer flow of a moving vertical plate with suction and dissipation in presence of magnetic field. In the present paper, motivated by works mentioned above and practical applications, an analysis is made to study the problem of two dimensional MHD boundary layer forced convective flow and heat transfer of an incompressible fluid past a plate embedded in a Darcy-Forchheimer porous. Thermal radiation term is considered in the energy equation. Using similarity transformation, the governing partial differential equations are transformed into a set of self-similar ordinary differential equations, which are then solved using Runge-Kutta fourth order method along with shooting technique. The numerical results are plotted in some figures and the variations in physical characteristics of the flow dynamics and heat transfer for several parameters involved in the equations are discussed.

FORMULATION OF THE PROBLEM

Consider a forced convective, two-dimensional steady lami-

nar boundary-layer flow of a viscous, incompressible, radiating, electrically conducting fluid over a flat plate of very small thickness and much larger breadth, embedded in a porous medium. The x-coordinate is measured along the plate from its leading edge and the y-coordinate is normal to it. A magnetic field B_0 of uniform strength is applied transversely to the direction of the flow. In the analysis of flow in porous media, the differential equation governing the fluid motion is based on Darcy-Forchheimer model, which accounts for the drag (represented by the Darcy term) exerted by the porous media as well as the inertia effect (represented by the non-linear Forchheimer term). The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. Taking into account the thermal radiation term and under the usual boundary layer approximations, the governing equations that are based on balance laws of mass, linear momentum and energy (Pai [35], Schlichting [36], Bansal [37]) under the influence of externally imposed transverse magnetic field (Jeffery [38], Bansal [39]) for this investigation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{v}{k} (u - u_\infty) - \frac{k'}{\sqrt{k}} (u^2 - u_\infty^2) - \frac{\sigma_e B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \quad (3)$$

Accompanied by the boundary conditions:

$$y = 0: u = 0, v = 0, T = T_w$$

$$y \rightarrow \infty: u = u_\infty, T = T_\infty \quad (4)$$

The governing partial differential equations (1)–(3) can be reduced to ordinary differential equations by introducing the following transformation:

$$\eta = y \left(\frac{u_\infty}{v x} \right)^{1/2}, \Psi = (u_\infty v x)^{1/2} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

Where η is the similarity variable and Ψ is the stream function defined in the usual way as $u = \frac{\partial \Psi}{\partial y}$, $v = -\frac{\partial \Psi}{\partial x}$ which identically satisfies (1).

The transformed non-linear ordinary differential equations are:

$$f''' + \frac{1}{2} f f'' - k_1 (f' - 1) - k_2 (f^2 - 1) + M f' = 0 \quad (6)$$

$$\frac{\theta''}{\Pr} \left[1 + \frac{4}{3N} \{1 + (\theta_r - 1)\theta\}^3 \right] + \frac{1}{\Pr N} \{1 + (\theta_r - 1)\theta\}^2 (\theta_r - 1) \theta^2 + \frac{1}{2} \theta \theta' = 0 \quad (7)$$

Where,

$$k_1 = \frac{1}{D_{k_1} Re_k} = \frac{v}{k_0 u_\infty} \text{ (Porous medium parameter)}, k_2 = \frac{k_0}{\sqrt{k_0}} \text{ (Inertial parameter)}$$

$$M = \frac{x \sigma_0 B_0^2}{\rho u_\infty} \text{ (Magnetic parameter)}, \text{Pr} = \frac{\mu C_p}{k} \text{ (Prandtl number)}$$

$$N = \frac{\kappa k^*}{4 \sigma_0 T_\infty^3} \text{ (Radiation parameter)}$$

$$\theta_r = \frac{T_w}{T_\infty} \text{ (Temperature ratio of the medium)}$$

The corresponding boundary conditions are reduced to:

$$\begin{aligned} f(0) &= S, f'(0) = 0, \theta(0) = 1 \\ f'(\infty) &\rightarrow 1, \theta(\infty) \rightarrow 0. \end{aligned} \quad (8)$$

DISCUSSIONS

The system of governing equations (6)-(7) together with the boundary condition (8) is non-linear ordinary differential equations depending on the various values of the physical parameters viz. Porous medium parameter k_1 , the Inertial parameter k_2 , the Magnetic parameter M , the Prandtl number Pr , and the Radiation parameter N . The system of equations (6)-(7) is solved by Runge-Kutta fourth order scheme with a systematic guessing of $f'(0)$ and $\theta'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.01$ is used while obtaining the numerical solution and accuracy upto the seventh decimal place i.e. 1×10^{-4} , which is very sufficient for convergence. The computations were done by a program which uses a symbolic and computer language Matlab.

RESULTS

Figure 1 shows the effect of porosity parameter (k_1) on the velocity profile. From this plot it is observed that the effect of increasing values of porosity parameter is to increases the velocity distribution in the flow region. Accordingly, the thickness of the velocity boundary layer decreases. In this case, horizontal velocity is found to increase with the increasing values of the permeability of the medium. With a rise in permeability of the medium, the Darcian body force decreases in magnitude (as it is inversely proportional to the permeability) Darcian resistance acts to decelerate the fluid particles in continua. This resistance diminishes as permeability of the medium increases. So progressively less drag is experienced by the flow and flow retardation is thereby decreased. Hence the velocity of the fluid increases as porosity parameter increases.

Figure 2 shows the effect of inertial parameter (k_2) on velocity components of fluid velocity. From this plot it is observed that the effect of increasing values of inertial parameter is to increases the velocity distribution i.e., the thickness of the

velocity boundary layer decreases. Thus, the non-Darcian term (i.e. Forchheimer term) has a very significant effect on the velocity profile.

The impact of the magnetic parameter (M) on the fluid flow is very significant in practical point of view. In Figure 3, the variation in velocity distribution for several values of magnetic parameter is presented. The dimensionless velocity $f'(\eta)$ increases with increasing values of magnetic parameter. Accordingly, the thickness of the velocity boundary layer decreases. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. To reduce velocity boundary layer thickness the generated Lorentz force enhances the fluid motion in the boundary layer region.

In Figures 4 and 5, the effect of suction parameter (S) are presented for non-porous ($k_1=0=k_2$) and porous media ($k_1=0.1=k_2$) respectively. We infer from these figures that the horizontal velocity increases with increase in suction parameter i.e. suction causes to increase the velocity of the fluid. Since the effect of suction is to suck away the fluid near the wall, the velocity boundary layer is reduced due to suction S ($S>0$). Consequently the velocity increases. Hence the velocity gradient increases with increasing values of suction parameter S ($S>0$).

Figures 6 and 7 are plotted for the temperature profiles for different values of permeability parameter (k_1) and inertial parameter (k_2) respectively for the given values of N , θ_r , M , Pr and S . We observed from both the figures that temperature decreases with increase of permeability parameter and inertial parameter within the boundary layer.

Figure 8, which illustrate the effect of magnetic parameter (M) on the temperature profile. We infer from this figure that the temperature decreases with an increase in magnetic parameter. In this case temperature asymptotically approaches to zero in the free stream region.

Figure 9 (in non-porous media) and Figure 10 (in porous media), which are the graphical representation of the temperature profiles for different values of the radiation parameter N for the given values of the parameters k_1 , k_2 , θ_r , M , Pr and S . We infer from these figures that temperature of the fluid decreases with increase in radiation parameter. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases as the thermal boundary layer thickness becomes thinner.

Figure 11 and 12 depict the effect of suction parameter S when $k_1=0=k_2$ and $k_1=0.1=k_2$ respectively for the given values of N , M , θ_r and Pr . We infer from these figures that temperature of the fluid decreases with increase in suction parameter in both the cases. The thermal boundary layer thickness decreases with the suction parameter which causes an increase in the rate of heat transfer. The reason of this

behavior is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness.

The effect of temperature ratio parameter θ_r on the temperature profiles are presented in Figures 13 and 14, in the case of non-porous media and porous media respectively. We observed from both the figures that the fluid temperature increases with increase of θ_r . $\theta_r > 1$ when $T_w > T_\infty$ and this is the usual case that is considered in this problem, $\theta_r \leq 1$ represents the case when $T_w \leq T_\infty$ and this is possible if the porous plate is kept on ice pad. Figure 15, which is a representation of the local dimensionless coefficient of heat transfer $-N(0)$, known as the Nusselt number for different values of Prandtl number (Pr) versus radiation parameter (N) for the given values of θ_r , M, S. We observed from this figure that the rate of heat transfer increases with the value of Pr.

CONCLUSION

A mathematical model has been presented for the MHD boundary layer forced convective flow and heat transfer of an incompressible fluid past a plate embedded in a Darcy-Forchheimer porous medium. The governing partial differential equations are converted into ordinary differential equations by using similarity transformations. The effect of several parameters controlling the velocity and temperature profiles are shown graphically and discussed briefly. The influence of the physical parameters viz. Porous medium parameter k_1 , the Inertial parameter k_2 , the Magnetic parameter M , the Prandtl number Pr , and the Radiation parameter N on dimensionless velocity and temperature profiles were examined. From the study, following conclusion can be drawn:

1. The effect of permeability of the medium on a viscous incompressible fluid is to increase the fluid velocity by reducing the drag on the flow which in turn causes a decrease in the temperature field.
2. It is found that the inertial parameter has a great influence on decreasing the flow field, whereas its influence is reversed on the rate of heat transfer. The non-Darcian term (i.e. Forchheimer term) has a very significant effect on the velocity distribution.
3. As the magnetic parameter M increases, we can find the velocity profile increases in the flow region and to decrease the temperature profile. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. Thus we conclude that we can control the velocity field and temperature by introducing magnetic field.
4. The velocity gradient increases with increasing values of suction parameter S . Since the effect of suction is to suck away the fluid near the wall, the velocity boundary layer is reduced due to suction. Consequently the velocity increases. Hence the velocity gradient in-

creases with increasing suction.

5. The temperature in the boundary layer decreases due to suction. Due to thermal radiation, temperature is found to decrease. This effect is more pronounced in presence of porous media. The combined effects of suction and thermal radiation can be used as means of cooling.
6. The fluid temperature increases with increase of temperature ratio parameter θ_r .
7. The rate of heat transfer increases with the increasing values of Prandtl number. This heat transfer is very important in production engineering to improve the quality of the final product.

ACKNOWLEDGEMENTS

Authors acknowledge the immense help received from the scholars whose articles are cited and included in references of this manuscript. The authors are also grateful to authors/editors/publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed.

Conflict of Interest: None

Source of Funding: One of the authors (D.K.) is grateful to the UGC for providing financial support in the form of BSR Fellowship, India.

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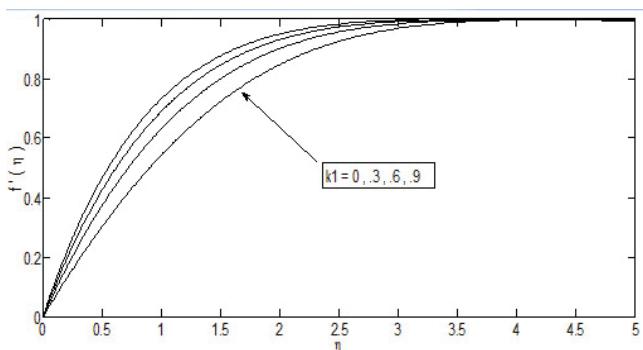


Figure 1: Velocity profile for various values of k_1 when $M=.01$, $N=1$, $Pr=1$, $S=.5$, $\theta_r = 1.1$, $k_2 = 0.1$.

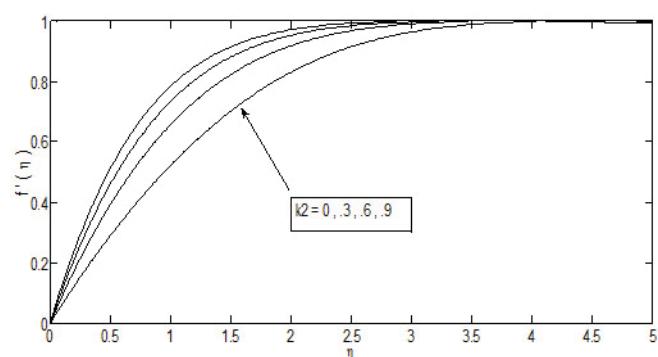


Figure 2: Velocity profile for various values of k_2 when $M=.01$, $N=1$, $Pr=1$, $S=.5$, $\theta_r = 1.1$, $k_1 = 0.1$.

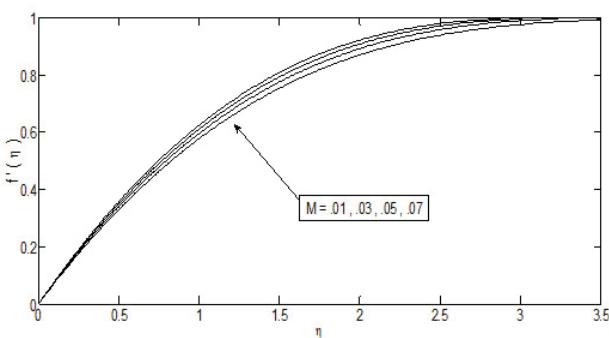


Figure 3: Velocity profile for various values of M when $N=1$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_1=0.1$, $k_2=0.1$.

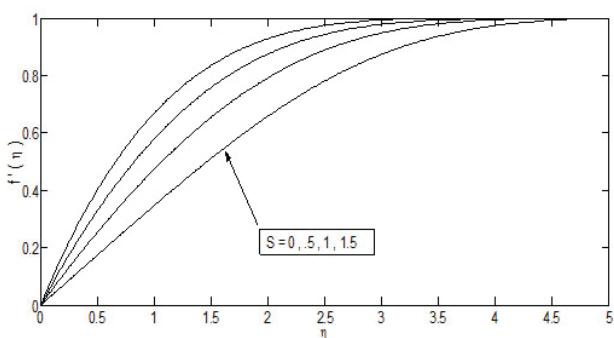


Figure 4: Velocity profile for various values of S when $N=1$, $Pr=1$, $M=.01$, $\theta_r=1.1$, $k_1=0$, $k_2=0$.

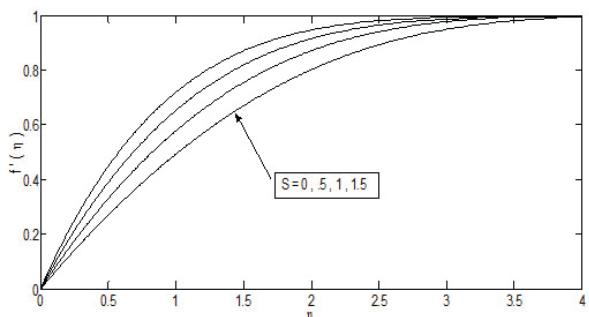


Figure 5: Velocity profile for various values of S when $N=1$, $Pr=1$, $M=.01$, $\theta_r=1.1$, $k_2=0.1$, $k_2=0.1$.

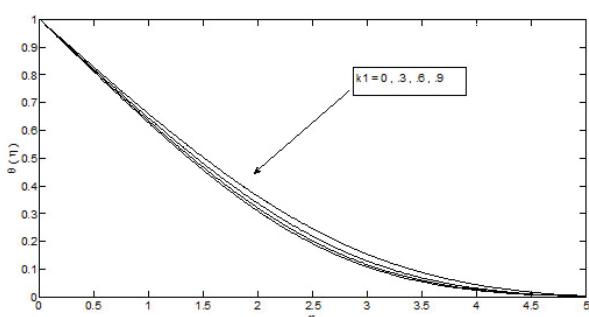


Figure 6: Temperature profile for various values of k_1 when $M=.01$, $N=1$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_2=0.1$.

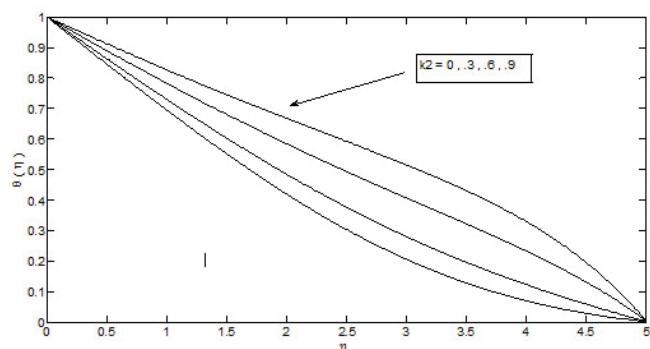


Figure 7: Temperature profile for various values of k_2 when $M=.01$, $N=1$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_1=0.1$.

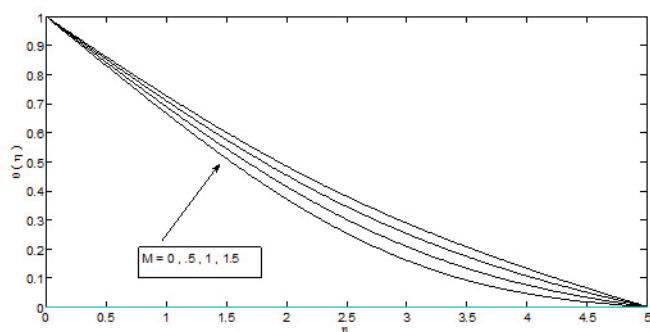


Figure 8: Temperature profile for various values of M when $N=1$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_1=0.1$, $k_2=0.1$.

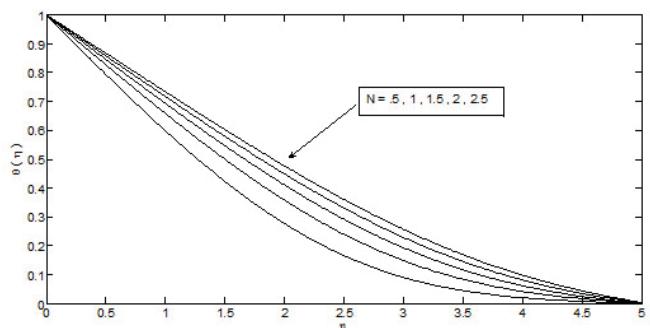


Figure 9: Temperature profile for various values of N when $M=.01$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_1=0$, $k_2=0$.

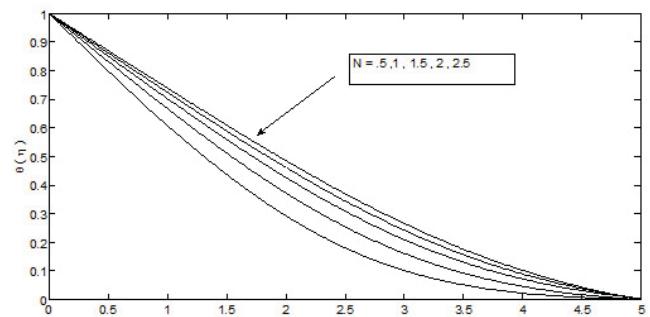


Figure 10: Temperature profile for various values of N when $M=.01$, $Pr=1$, $S=.5$, $\theta_r=1.1$, $k_1=0.1$, $k_2=0.1$.

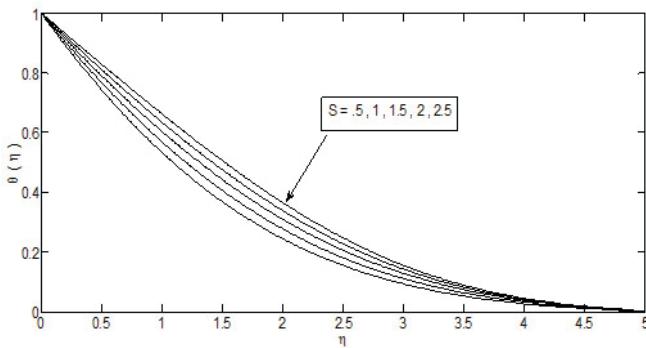


Figure 11: Temperature profile for various values of S when $M=.01$, $Pr=1$, $N=1$, $\theta_r=1.1$, $k_1=0$, $k_2=0$

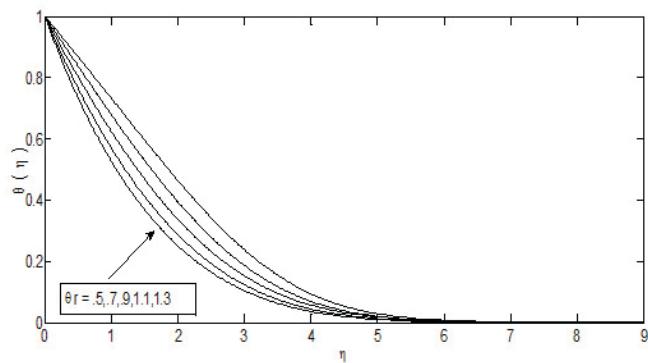


Figure 14: Temperature profile for various values of θ_r when $M=.01$, $Pr=1$, $N=1$, $S=.5$, $k_1=0.1$, $k_2=0.1$

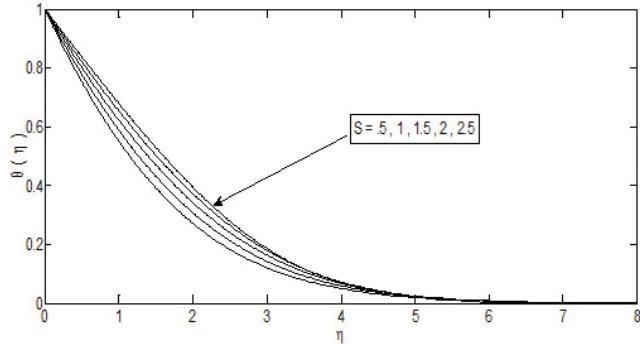


Figure 12: Temperature profile for various values of S when $M=.01$, $Pr=1$, $N=1$, $k_1=1.1$, $k_1=0.1$, $k_2=0.1$

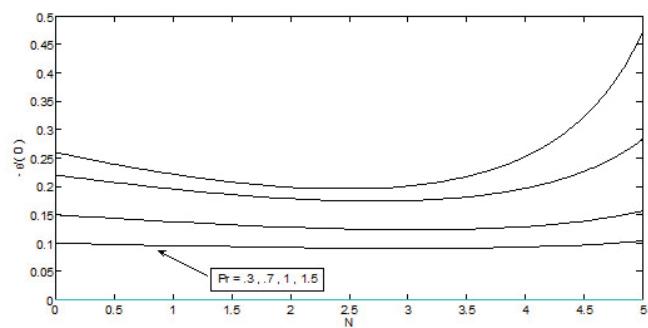


Figure 15: Variation of $-\theta'(0)$ with the Radiation parameter N for various values of Pr when $M=.01$, $S=.5$, $\theta_r=1.1$, $k_1=0$, $k_2=0$.

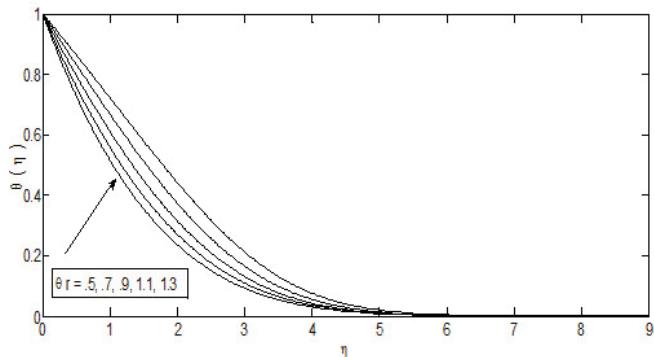


Figure 13: Temperature profile for various values of k_1 when $M=.01$, $Pr=1$, $N=1$, $S=.5$, $k_1=0$, $\theta_r=0$.